ME 423: FLUIDS ENGINEERING

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Lecture-06-07 (21/09/2024)

Hydraulics of Pipeline Systems

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Pipeline systems

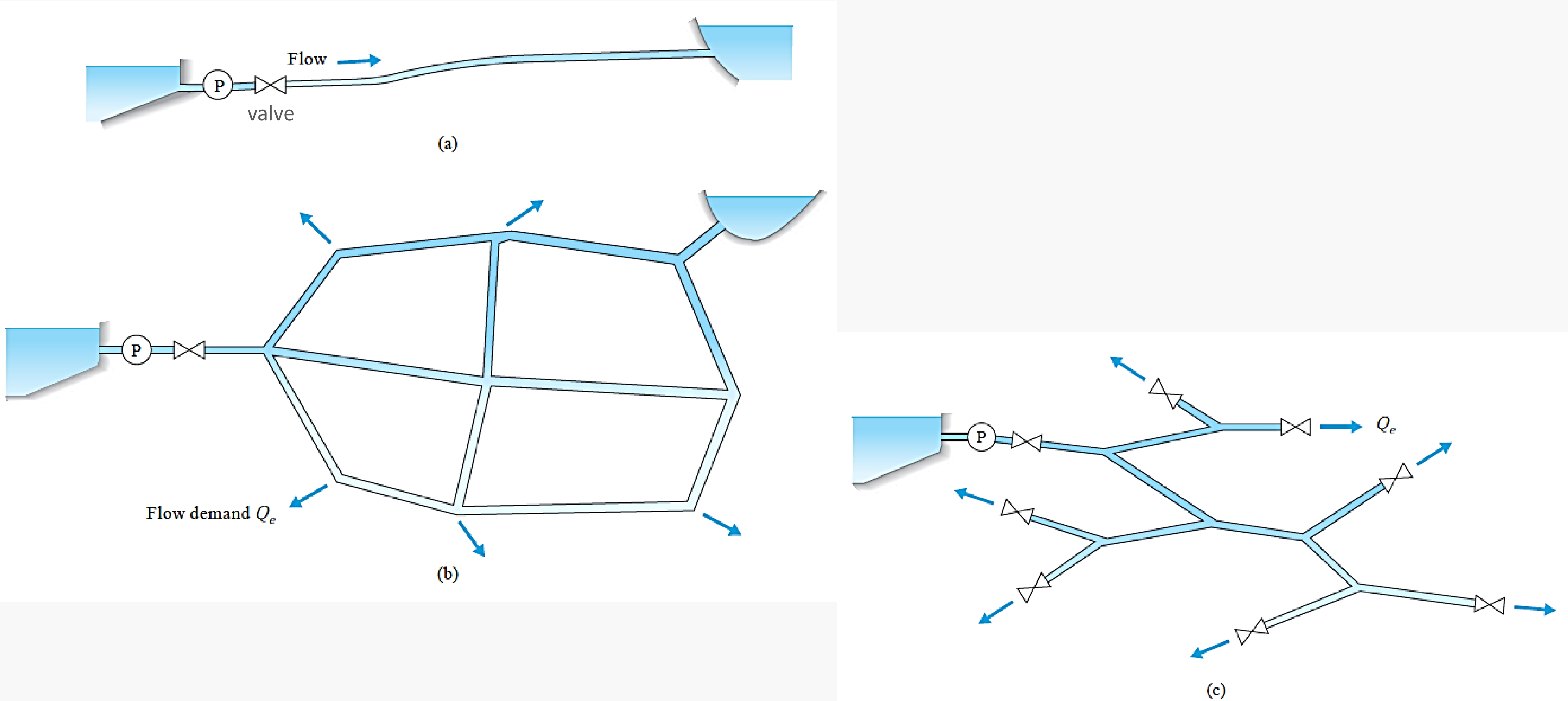


Fig. 11.1 Pipe systems: (a) single pipe; (b) distribution network; (c) tree network.

Frictional Losses in pipeline systems

It is convenient to express the pipe element frictional loss in the exponential form

$$h_L = RQ^n$$

in which h_L is the head loss over length L of pipe, R is the resistance coefficient, Q is the discharge in the pipe, and n is an exponent. Comparing the Darcy - Weisbach relation with this expression, one gets n = 2, and the resulting expression for the **resistance coefficient denoted by *R* is**

$$R = \frac{fL}{2gDA^2} = \frac{8fL}{g\pi^2 D^5}$$

where f is the friction factor and may be determined from the Moody diagram or various correlations.

Frictional Losses in pipeline systems

Swamee and Jain (1976)

$$f = 1.325\left\{\ln\left[0.27\left(\frac{e}{D}\right) + 5.74\left(\frac{1}{\mathrm{Re}}\right)^{0.9}\right]\right\}^{-2}$$

Combining the above two equations, one finds that,

$$\boxed{R = 1.07\left(\frac{L}{gD^5}\right)\left\{\ln\left[0.27\left(\frac{e}{D}\right) + 5.74\left(\frac{1}{\mathrm{Re}}\right)^{0.9}\right]\right\}^{-2}}$$

Above equations are valid over the ranges $0.01 > e/D > 10^{-8}$, $10^8 > \mathrm{Re} > 5000$.
The fully rough regime, where Re has a negligible effect on *f*, begins at a Reynolds number given by,

$$\mathrm{Re} = \frac{200D}{e\sqrt{f}}$$

For the values of Re greater than this, the friction factor is a function only of e/D, and is given by,

$$\boxed{f = 1.325\left\{\ln\left[0.27\left(\frac{e}{D}\right)\right]\right\}^{-2}}$$

Series Piping

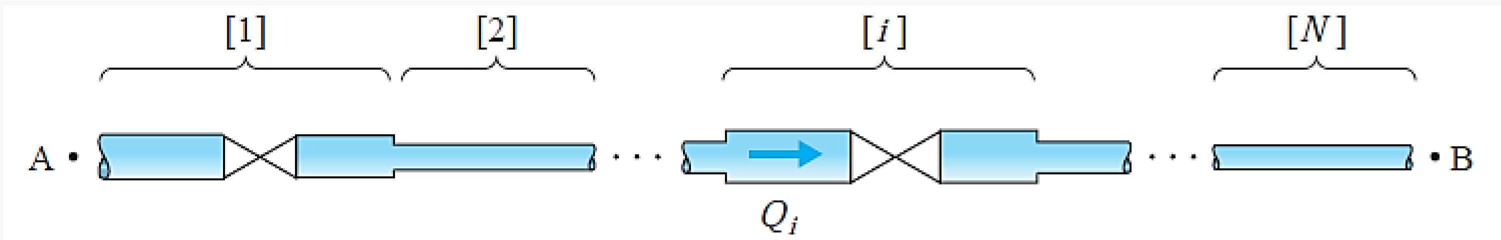
Consider the series system shown below. It consists of N pipe elements with a specified number of minor-loss components ΣK associated with each ith pipe element.

A single minor loss is equal to $h_L = KV^2/2g = KQ^2/2gA^2$.

For many flow situations, it is common practice to neglect the kinetic-energy terms at the inlet and outlet; they would be significant only if the velocities were relatively high.

So, the energy equation applied from location A to location B is

$$\left(\frac{p}{\gamma}+z\right)_{\mathrm{A}}-\left(\frac{p}{\gamma}+z\right)_{\mathrm{B}}=\left(R_1+\frac{\Sigma K}{2gA_1^2}\right)Q_1^2+\left(R_2+\frac{\Sigma K}{2gA_2^2}\right)Q_2^2+\cdots+\left(R_N+\frac{\Sigma K}{2gA_N^2}\right)Q_N^2=\sum_{i=1}^{N}\left(R_i+\frac{\Sigma K}{2gA_i^2}\right)Q_i^2$$



Series Piping

in which R_i is the resistance coefficient for pipe i. The statement of continuity for the series system is that the discharge in every element is identical, or

$$Q_1 = Q_2 = \cdot \cdot \cdot = Q_i = \cdot \cdot \cdot = Q_N = Q$$

$$\left(\frac{p}{\gamma} + z\right)_A - \left(\frac{p}{\gamma} + z\right)_B = \left(R_1 + \frac{\Sigma K}{2gA_1^2}\right) Q_1^2 + \left(R_2 + \frac{\Sigma K}{2gA_2^2}\right) Q_2^2 + \cdot \cdot \cdot + \left(R_N + \frac{\Sigma K}{2gA_N^2}\right) Q_N^2 = \sum_{i=1}^{N} \left(R_i + \frac{\Sigma K}{2gA_i^2}\right) Q_i^2$$

Replacing Q_i with Q

$$\left(\frac{p}{\gamma} + z\right)_A - \left(\frac{p}{\gamma} + z\right)_B = \left[\sum_{i=1}^{N} \left(R_i + \frac{\Sigma K}{2gA_i^2}\right)\right] Q^2$$

Problem

Example 11.2

For the system shown in Fig. E11.2, find the required power to pump 100 L/s of liquid ($S = 0.85$, $\nu = 10^{-5}$ m^2/s). The pump is operating at an efficiency $\eta = 0.75$. Pertinent data are given in the figure.

Line 1: $L = 10$ m, $D = 0.20$ m, $e = 0.05$ mm, $K_1 = 0.5$, $K_v = 2$
Line 2: $L = 500$ m, $D = 0.25$ m, $e = 0.05$ mm, $K_e = 0.25$, $K_2 = 1$

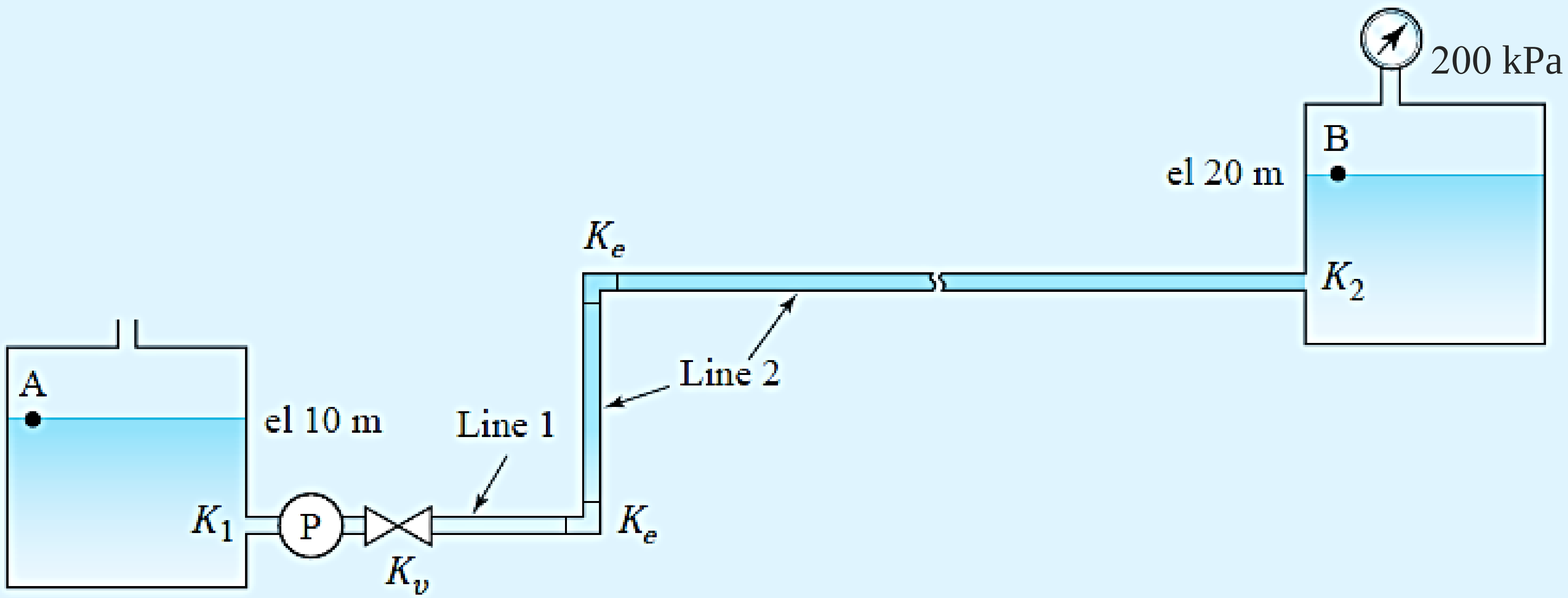


Fig. E11.2

(continued)

Solution

This is a category 1 problem. The energy relation, Eq. 11.3.3, for the system is

$$\left(\frac{p}{\gamma}+z\right)_A + H_P = \left(\frac{p}{\gamma}+z\right)_B + \left[R_1 + \frac{K_1+K_v}{2gA_1^2} + R_2 + \frac{2K_e+K_2}{2gA_2^2}\right]Q^2$$

The resistance coefficients, R_1 and R_2, are calculated with Eq. 11.2.4 after first evaluating Re and e/D:

$$\mathrm{Re}_1 = \frac{4Q}{\pi D_1 \nu} = \frac{4\times 0.10}{\pi \times 0.20 \times 10^{-5}} = 6.37\times 10^4 \qquad \left(\frac{e}{D}\right)_1 = \frac{0.05}{200} = 0.00025$$

$$\mathrm{Re}_2 = \frac{4Q}{\pi D_2 \nu} = \frac{4\times 0.10}{\pi \times 0.25 \times 10^{-5}} = 5.09\times 10^4 \qquad \left(\frac{e}{D}\right)_2 = \frac{0.05}{250} = 0.0002$$

$$R_1 = 1.07\left(\frac{10}{9.81\times 0.20^5}\right)$$

$$\times \{\ln[0.27\times 0.00025 + 5.74\times(6.37\times 10^4)^{-0.9}]\}^{-2} = 53.4$$

$$R_2 = 1.07\left(\frac{500}{9.81\times\ 0.25^5}\right)$$

$$\times \{\ln[0.27\times 0.0002 + 5.74\times(5.09\times 10^4)^{-0.9}]\}^{-2} = 904$$

The minor loss coefficient terms are calculated to be

$$\frac{K_1+K_v}{2gA_1^2} = \frac{0.5+2}{2\times 9.81\times[\pi/4\times 0.20^2]^2} = 129.1$$

$$\frac{2K_e+K_2}{2gA_2^2} = \frac{2\times 0.25+1}{2\times 9.81\times[\pi/4\times 0.25^2]^2} = 31.7$$

$$R = 1.07\left(\frac{L}{gD^5}\right)\left\{\ln\left[0.27\left(\frac{e}{D}\right) + 5.74\left(\frac{1}{\mathrm{Re}}\right)^{0.9}\right]\right\}^{-2}$$

Substitute these values into the energy equation and obtain

$$0 + 10 + H_P = \frac{200 \times 10^3}{0.85 \times 9800} + 20 + (53.4 + 129.1 + 904 + 31.7)\, 0.1^2$$

This relation reduces to

$$10 + H_P = 24 + 20 + 11.2$$

Solving for the head across the pump gives $H_P = 45.2$ m. The required input power is

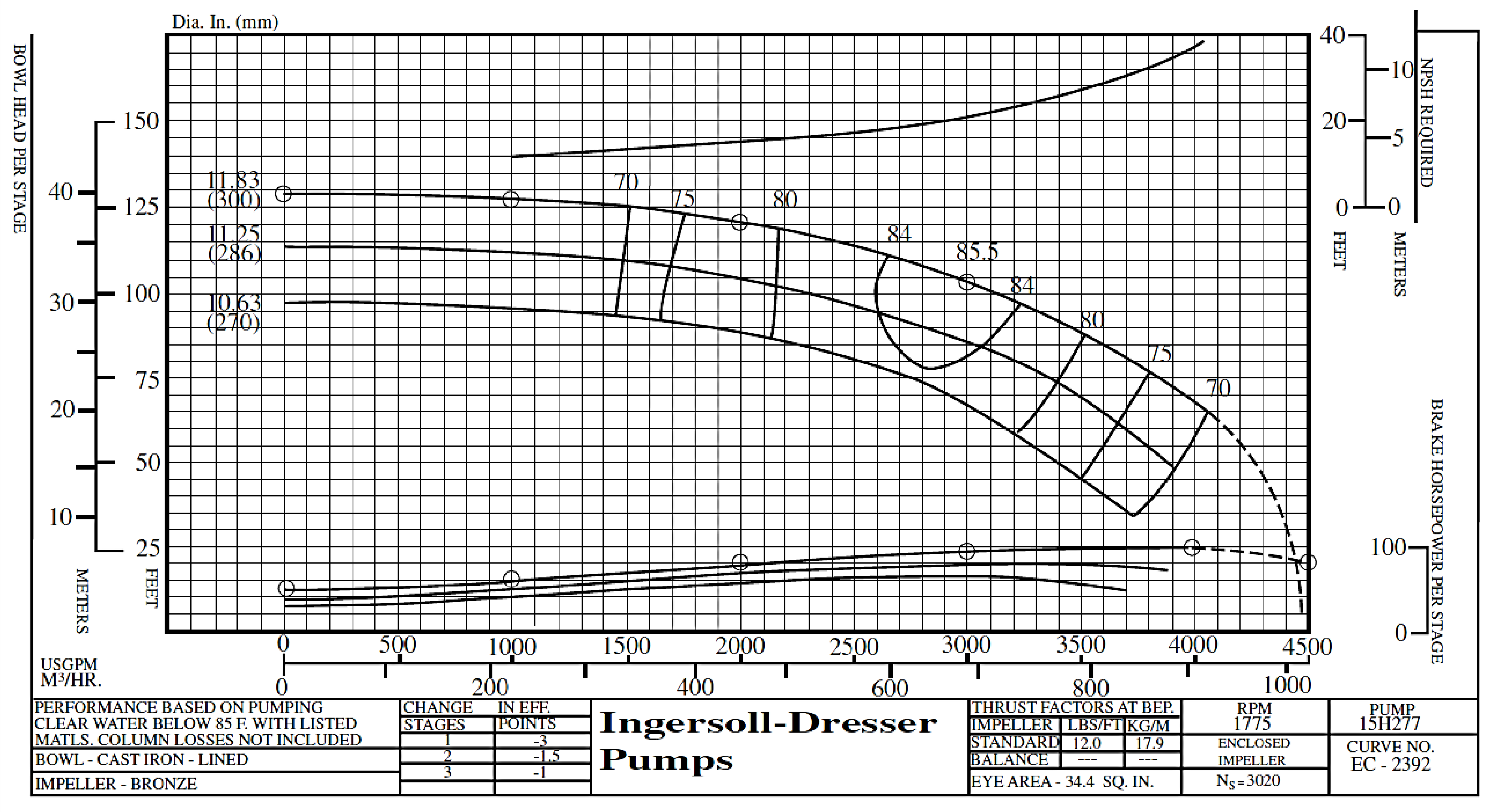
$$\dot{W}_P = \frac{\gamma Q H_P}{\eta}$$

$$= \frac{(9800 \times 0.85) \times 0.10 \times 45.2}{0.75}$$

$$= 5.0 \times 10^4 \text{ W} \qquad \text{or} \qquad 50 \text{ kW}$$

SERIES PIPE FLOW WITH PUMP(S)

Pump characteristics curve



SERIES PIPE FLOW WITH PUMP(S)

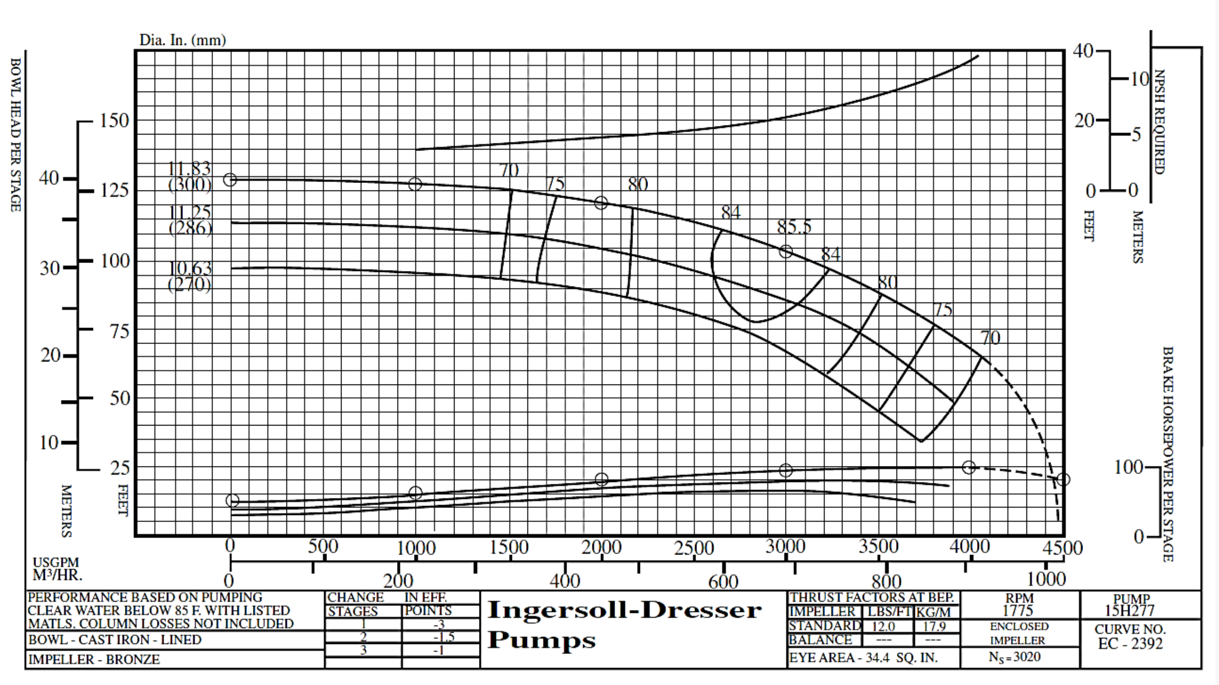
The solution of pipe flow problems involving pumps normally requires us to read data from pump characteristic curves.

Let the pump characteristic curve for the head h_p be expressed by a second-order polynomial

$$h_p = AQ^2 + BQ + C$$

in which the coefficients A, B, and C are determined by the use of three (h_p, Q) data pairs that bracket the expected range of operation of the pump. To obtain the coefficients, we write three equations by substituting each data pair into the polynomial to obtain

$$AQ_1^2 + BQ_1 + C = h_{p1}$$
$$AQ_2^2 + BQ_2 + C = h_{p2}$$
$$AQ_3^2 + BQ_3 + C = h_{p3}$$

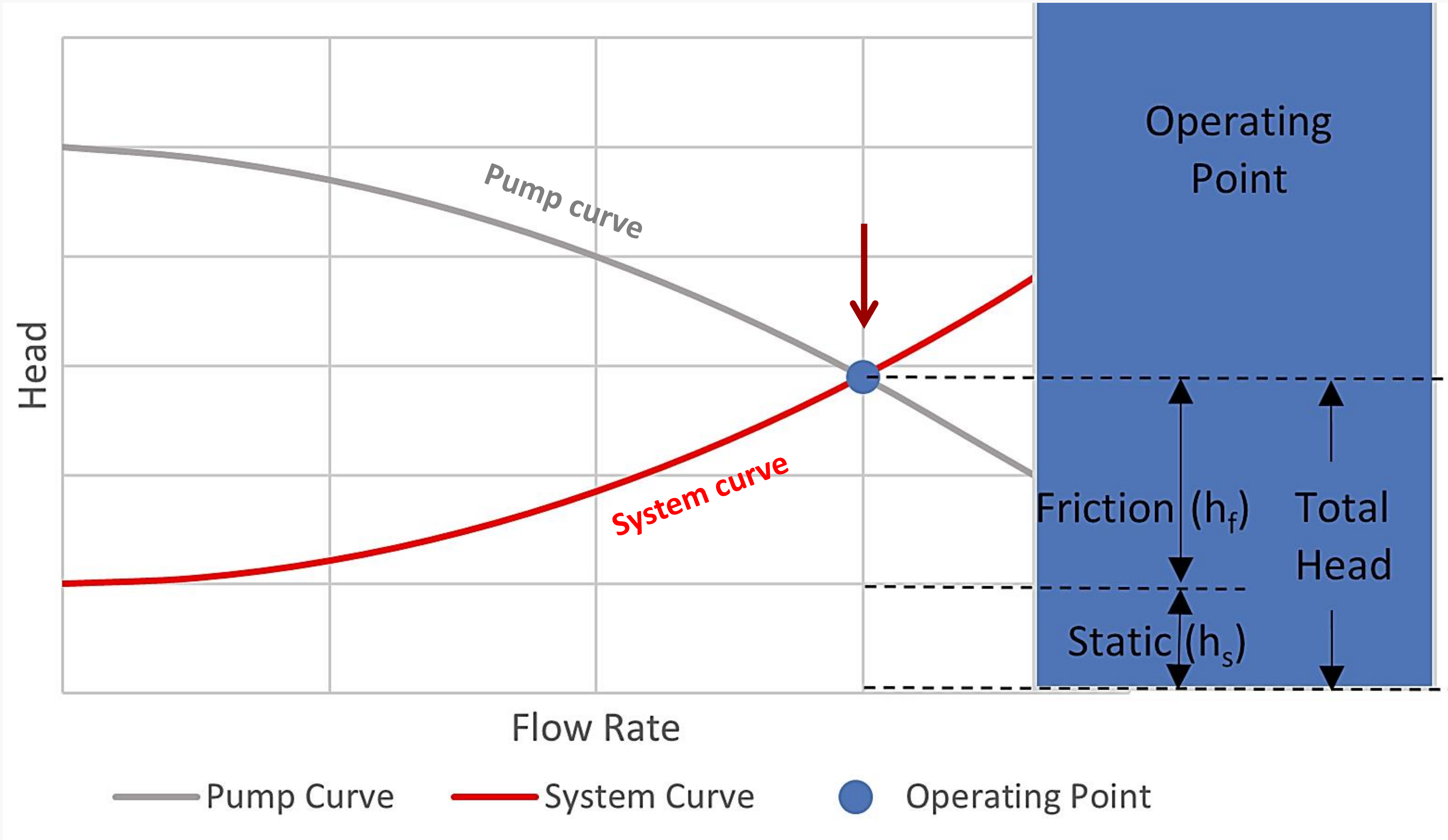


In matrix notation the above equation comes as,

$$\begin{bmatrix} Q_1^2 & Q_1 & 1 \\ Q_2^2 & Q_2 & 1 \\ Q_3^2 & Q_3 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} h_{p1} \\ h_{p2} \\ h_{p3} \end{Bmatrix}$$

which can be solved in various ways to determine the coefficients.

System curve and Pump characteristics curve

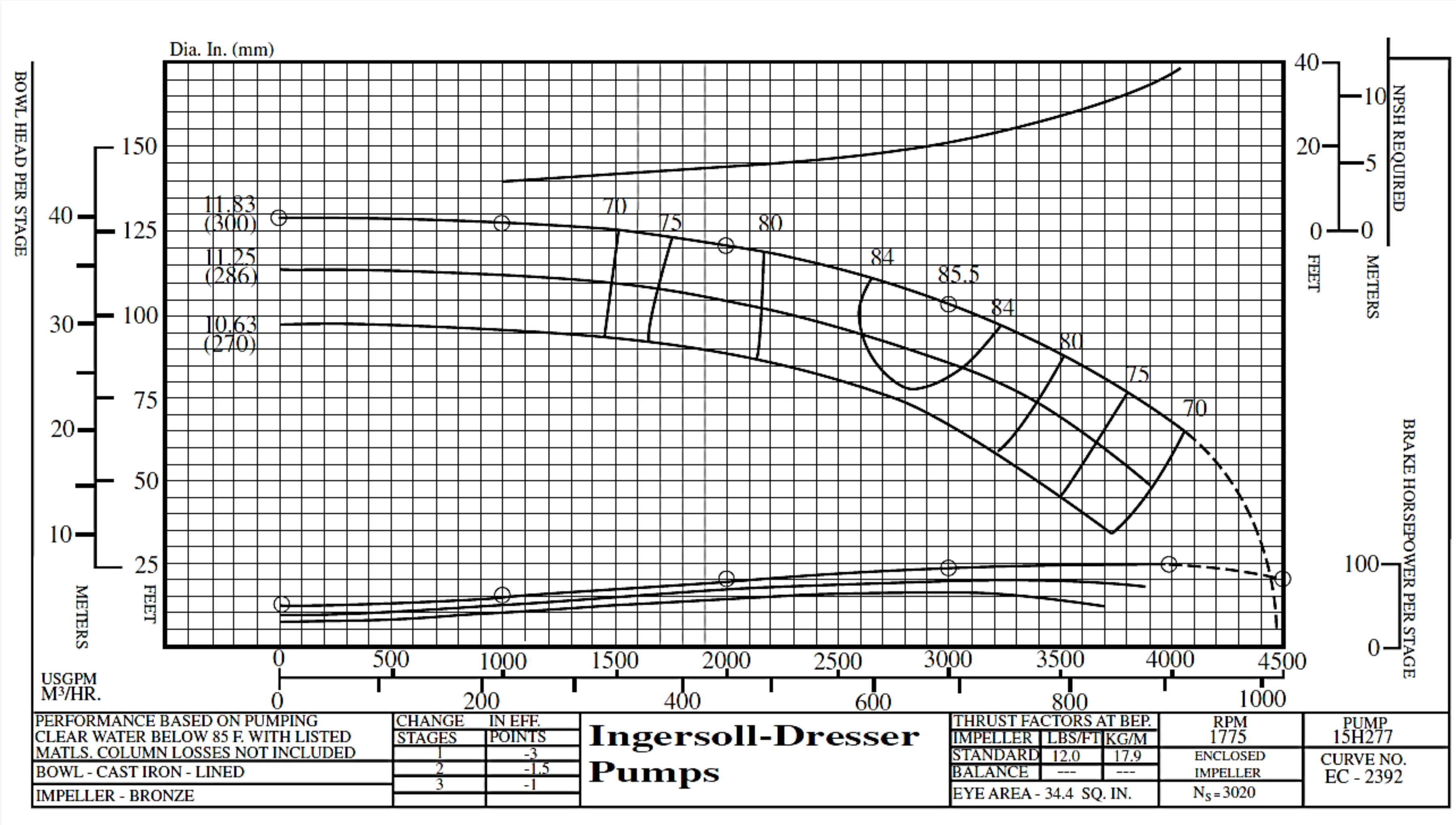


(ME 421: Fluid Machinery)

Problem

(Example Problem 2.4)

A single-stage Ingersoll-Dresser 15H277 pump, outfitted with the largest impeller (Refer to the pump characteristic curve shown below), is used to pump water from a reservoir at elevation 1350 ft to another reservoir at elevation 1425 ft. The line is 6000 ft long and 18 in. in diameter with an equivalent sand grain roughness e = 0.015 in. ($\nu = 1.14 \times 10^{-5}$ ft^2/s). Neglecting local losses, compute the discharge in the pipeline.



Solution:

Apply work-energy equation between the two reservoir water surfaces:

$$1350 = 1425 + h_f - h_p$$

or

$$h_p = 75 + f\frac{L}{D}\frac{Q^2/A^2}{2g} = 75 + f\frac{6000}{1.5}\frac{Q^2}{2g(1.767)^2} = 75 + 19.9fQ^2$$

$$\boxed{h_p = 75 + 19.9fQ^2}$$

from pump System req.

There are three unknowns in this equation: h_p, Q, and f. They must be determined by using this equation, the pump curve and the Colebrook-White equation. We shall obtain the solution in two ways, first by hand and then with the aid of a computer.

The hand solution begins by (a) selecting a trial discharge, (b) solving the Colebrook-White equation, Eq. 2.12, for f, (c) calculating h_p from the above work-energy equation, (d) comparing this h_p with the value that is read from the pump characteristic curve, and (e) repeating the process until the h_p's match, as summarized in the table:

Use **1 ft³/s = 448.8 US GPM** **1 US Galon = 231 in³ = 0.1336 ft³**

Colebrook- White equation

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log_{10}\left(\frac{e}{D} + \frac{9.35}{Re\sqrt{f}}\right)$$

$$\boxed{h_p = 75 + 19.9 f Q^2}$$

from pump System req.

(a) Q gal/min	Q ft^3/s	(b) f	(c) h_p ft	(d) hp, curve ft
3000	6.68	0.01961	92.4	103
3500	7.80	0.01950	98.6	88
3300	7.35	0.01951	96.0	95
3280	7.31	0.01954	95.8	95.8

You need to go with detail calculations.

The discharge is 3280 gal/min by this method.

Another Approach:

The pump curve must be defined by an algebraic equation if the computer is to be used in solving for h_p, Q, and f. A second-order polynomial can be fit to the Ingersoll-Dresser 15H277 pump curve by applying Eqs. 2.33 and 2.34 and using the three data pairs (103.0, 6.68), (95.0, 7.35), and (88.0, 7.80). Equation 2.34 gives the matrix form of this problem as

$$\begin{bmatrix} Q_1^2 & Q_1 & 1 \\ Q_2^2 & Q_2 & 1 \\ Q_3^2 & Q_3 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{bmatrix} 44.62 & 6.68 & 1 \\ 54.02 & 7.35 & 1 \\ 60.84 & 7.80 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 103 \\ 95 \\ 88 \end{Bmatrix}$$

yielding a solution $A = -3.224$, $B = 33.293$, and $C = 24.472$ so that the pump curve is approximately

$$h_p = -3.224Q^2 + 33.293Q + 24.472$$

Solution:

hp = 95.7 ft, Q = 7.30 ft3/s = 3280 gal/min, and f = 0.019546